## Solutions to Linear Equations

## Question 1 .

Directions: Type the correct answer in each box. Use numerals instead of words. If necessary, use / for the fraction bar(s).
Complete the equation below to create an equation with infinitely many solutions.

$$
7 x-8=8+\square x-\square+3 x
$$

Question 2.
Which of the following equations has infinitely many real solutions?
A. $3(x-6)=3 x+18$
B. $3(x-6)=3 x-18$
C. $3 x+1=x-1$
D. $3 x+1=3 x-1$

Question 3 .
Which of the following equations has no real solutions?
A. $2 x-4=8 x-4$
B. $2(x-4)=2 x-8$
C. $2 x-4=2 x-4$
D. $2(x+4)=2(x+15)$

Question 4 .
Which of the following best describes the solution to the equation below?

$$
26 x+24=-39 x-8
$$

A. infinite real solutions
B. no real solutions
C. exactly one real solution, $x=-\frac{32}{65}$
D. exactly one real solution, $x=2 \frac{6}{13}$

Question 5.
Which statement represents the simplified form of the given equation and correctly describes the solution?

$$
12 x-1=2(6+5 x)+7
$$

A. $x=20$; exactly one real solution
B. $x=10$; exactly one real solution
C. $\quad 1 \neq 19$; no real solution
D. $19=19$; infinite real solutions

## Question 6.

Which of the following best describes the solution to the equation below?

$$
6+2 x=\frac{1}{2}(12+4 x)
$$

A. no real solutions
B. infinite real solutions
C. exactly one real solution, $x=\frac{2}{3}$
D. exactly one real solution, $x=-4$

## Question 7.

Complete the equation below to create an equation with exactly one real solution.

$$
3(3 x+5)-4 x=\ldots x-10
$$

A. Cannot be determined
B. 5
C. -5

## Question 8 .

Michael hires a cab that charges a fare of $\$ 0.50$ per mile, plus an initial charge of $\$ 2$. Jason hires a cab that charges a fare of $\$ 0.50$ per mile, plus an initial charge of $\$ 3.50$.

At how many miles will the fares paid by Michael and Jason become equal?
A. The fares paid by Michael and Jason will never be equal.
B. The fares paid by Michael and Jason will be equal at every mile.
C. The fares paid by Michael and Jason will become equal at 6 miles.
D. The fares paid by Michael and Jason will become equal at 1.5 miles.

## Question 9 .

Which of the following equations has exactly one real solution?
A. $7(x+11)=7 x+77$
B. $7 x+11=7 x-18$
C. $7 x+11=-7 x-18$
D. $7 x+11=7 x+11$

## Question 10 .

Complete the equation below to create an equation with no solution.

$$
5(4 x+7)-2 x=\ldots x+21
$$

A. 18
B. -20
C. 20
D. -18

## Question 11

Which of the following best describes the solution to the equation below?

$$
7 x+4=7 x+4
$$

A. no real solutions
B. exactly one real solution, $x=4$
C. exactly one real solution, $x=\frac{4}{7}$
D. infinite real solutions

## Question 12 .

Erin and Dan went shopping at their local store. Erin bought shirts that cost $\$ 12$ each and spent $\$ 18$ on accessories. Dan bought the same number of shirts as Erin for $\$ 16$ each and spent $\$ 10$ on accessories.

If Erin and Dan were billed the same amount by the store, how many shirts did each of them buy?
A. Erin and Dan bought 4 shirts each.
B. Erin and Dan bought 2 shirts each.
C. Erin and Dan bought 3 shirts each.
D. Erin and Dan bought 1 shirt each.

Question 13.

## Directions: Drag each equation to the correct location on the model.

Classify each of the equations below according to the number of solutions.


## Question 14 .

## Directions: Select the correct answer from each drop-down menu.

Complete the equation so that it has exactly one solution.

$$
\frac{4}{3}(-9 x+12)-x=\square \Delta x-9
$$

Complete the equation so it has no solution.

$$
\frac{4}{3}(-9 x+12)-x=\square \Delta x+11
$$

Complete the equation so that it has infinitely many solutions.

$$
\frac{4}{3}(-9 x+12)-x=\square \hat{\nabla} x+\square \text { 㐪 }
$$

## Question 15.

Rick and Tom rented party halls. The Celebrations party hall charged Rick a rental fee of $\$ 65$, including music, and $\$ 25$ per guest. The Feast party hall charged Tom a rental fee of \$40, $\$ 25$ for music, and $\$ 25$ per guest.

If each of them spent the same amount of money, how many guests attended Rick and Tom's party?
A. The same number of guests attended Rick and Tom's party.
B. Five more guests attended Rick's party than Tom's party.
C. Fifteen more guests attended Tom's party than Rick's party.
D. Twenty-five more guests attended Rick's party than Tom's party.

## Answers

1. --
2. $B$
3. D
4. C
5. B
6. B
7. C
8. A
9. C
10. A
11. D
12. $B$
13. --
14. --
15. A

## Explanations

1. An equation with infinitely many solutions will be true for any value of $x$; so, it has equivalent expressions on both sides of the equal sign. Therefore, to create an equation with infinitely many solutions, make the right side of the equation equal to the left side of the equation. First, find the coefficient in front of the variable. Consider the terms containing an $x$-variable. On the left side of the equal sign, the expression has an $x$-term with a coefficient of 7 . Therefore, the term in the expression on the right side of the equal sign should also have a coefficient of 7 when simplified. Since there is a $3 x$ on the right side, the missing coefficient should be 4 .
Then, find the value of the constant. On the left side of the equation, the constant is -8 . So, the constant on the right side should simplify to -8 . Since there is an 8 on the right side, the missing constant is -16 .
Therefore, the equation with infinitely many solutions is below.

$$
7 x-8=8+\underline{4} x-\underline{16}+3 x
$$

2. Solve the following equation.

$$
\begin{aligned}
3(x-6) & =3 x-18 \\
3 x-3 \times 6 & =3 x-18 \\
3 x-3 x & =-18+18 \\
0 & =0
\end{aligned}
$$

Therefore, the equation $\mathbf{3}(\boldsymbol{x}-\mathbf{6})=\mathbf{3 x - 1 8}$ has infinitely many real solutions.
3. Solve the given equation.

$$
\begin{aligned}
2(x+4) & =2(x+15) \\
2 x+8 & =2 x+30 \\
2 x-2 x & =30-8 \\
0 & \neq 22
\end{aligned}
$$

Since 0 is never equal to 22 , no real number would satisfy the equation $2(x+4)=2(x+15)$.
Therefore, the equation $2(x+4)=2(x+15)$ has no real solutions.
4. Solve the given equation.

$$
\begin{aligned}
26 x+24 & =-39 x-8 \\
26 x+39 x & =-8-24 \\
65 x & =-32 \\
x & =-\frac{32}{65}
\end{aligned}
$$

Therefore, the equation $26 x+24=-39 x-8$
has exactly one real solution, $x=-\frac{32}{65}$.
5. Simplify the given equation.

$$
\begin{aligned}
12 x-1 & =2(6+5 x)+7 \\
12 x-1 & =12+10 x+7 \\
12 x-1 & =10 x+19 \\
2 x & =20 \\
x & =10
\end{aligned}
$$

So, the statement that represents the simplified form of the given equation and correctly describes the solution is given below.

$$
x=10 ; \text { exactly one real solution }
$$

6. Solve the given equation.

$$
\begin{aligned}
6+2 x & =\frac{1}{2}(12+4 x) \\
6+2 x & =6+2 x \\
6-6 & =2 x-2 x \\
0 & =0
\end{aligned}
$$

Therefore, the equation, $6+2 x=\frac{1}{2}(12+4 x)$, has infinite real solutions.
7. An equation has exactly one solution when it has different coefficients on both sides of the equation.

Therefore, to create an equation with exactly one solution, the coefficient of $x$ on the left side should not be equal to the coefficient of $x$ on the right side.

First, simplify the left side of the equation.

$$
\begin{aligned}
3(3 x+5)-4 x & =\_x-10 \\
9 x+15-4 x & =\_x-10 \\
5 x+15 & =\_x-10
\end{aligned}
$$

On the right side of the equation, the coefficient of $x$ can be any value other than 5 . From the given choices, -5 is the only value that will create an equation with exactly one real solution.

Therefore, an equation with exactly one solution is shown below.

$$
3(3 x+5)-4 x=-\underline{5} x-10
$$

8. Let $x$ be the number of miles

Michael was charged $\$ 0.50$ per mile plus $\$ 2$. Write an expression to model the fare paid by Michael for $x$ miles.

$$
0.5 x+2
$$

Jason was charged $\$ 0.50$ per mile plus $\$ 3.50$. Write an expression to model the fare paid by Jason for $x$ miles.

$$
0.5 x+3.5
$$

To find the miles at which the fares paid by Michael and Jason become equal, create an equation using the two expressions, and solve for $x$.

$$
\begin{aligned}
0.5 x+2 & =0.5 x+3.5 \\
0.5 x+2-0.5 x & =0.5 x+3.5-0.5 x \\
2 & \neq 3.5
\end{aligned}
$$

So, the above equation does not hold true for any value of $x$.

Therefore, the fares paid by Michael and Jason will not become equal at any point.
9. Solve the following equation.

$$
\begin{aligned}
7 x+11 & =-7 x-18 \\
7 x+7 x & =-18-11 \\
14 x & =-29 \\
x & =-2 \frac{1}{14}
\end{aligned}
$$

Therefore, the equation $\mathbf{7 x + 1 1 = - 7 x - 1 8}$ has exactly one real solution, at $x=-2 \frac{1}{14}$.
10. An equation has no solution when it has the same coefficient but different constants on both sides of the equation.

Therefore, to create an equation with no solution, the coefficient of $x$ on the left side should equal the coefficient of $x$ on the right side.
First, simplify the left side of the equation.

$$
\begin{aligned}
5(4 x+7)-2 x & =-x+21 \\
20 x+35-2 x & =-x+21 \\
18 x+35 & =-x+21
\end{aligned}
$$

Note that the constant terms on both sides of the given equation are different.
The coefficient of $x$ on the left side of the equation is 18 . To create an equation with no solution, the coefficient of $x$ on the right side must be 18 , as well.

Therefore, an equation with no solution is shown below.

$$
5(4 x+7)-2 x=\underline{18} x+21
$$

11. Solve the given equation.

$$
\begin{aligned}
7 x+4 & =7 x+4 \\
7 x-7 x & =4-4 \\
0 & =0
\end{aligned}
$$

Therefore, the equation $7 x+4=7 x+4$ has infinite real solutions.
12. Let $x$ be the number of shirts.

Erin bought $x$ shirts that cost $\$ 12$ each and spent $\$ 18$ on accessories. Write an expression to model this situation.

$$
12 x+18
$$

Dan bought $x$ shirts for $\$ 16$ each and spent $\$ 10$ on accessories. Write an expression to model this situation.

$$
16 x+10
$$

They were billed the same amount by the store. To find the number of shirts each of them bought, create an equation using the two expressions, and solve for $x$.

$$
\begin{aligned}
12 x+18 & =16 x+10 \\
8 & =4 x \\
2 & =x
\end{aligned}
$$

Therefore, Erin and Dan bought 2 shirts each.
13. Solve each equation.

$$
\begin{aligned}
5 x-20 & =3(x-4) \\
5 x-20 & =3 x-12 \\
2 x-20 & =-12 \\
2 x & =8 \\
x & =4
\end{aligned}
$$

Since $x=4$, then this equation has one real solution.

$$
\begin{aligned}
12 x+21 & =3(4 x+6) \\
12 x+21 & =12 x+18 \\
21 & =18
\end{aligned}
$$

Since 21 does not equal 18, this equation has no real solutions.

$$
\begin{aligned}
8(x+7) & =6(x-4) \\
8 x+56 & =6 x-24 \\
2 x+56 & =-24 \\
2 x & =-80 \\
x & =-40
\end{aligned}
$$

Since $x=-40$, this equation has one real solution.

$$
\begin{aligned}
5(x+8)-7 x & =-(2 x-4) \\
5 x+40-7 x & =-2 x+4 \\
-2 x+40 & =-2 x+4 \\
40 & =4
\end{aligned}
$$

Since 40 does not equal 4 , this equation has no real solutions.

$$
\begin{aligned}
24 x+12 & =2(12 x+6) \\
24 x+12 & =24 x+12 \\
12 & =12
\end{aligned}
$$

Since $12=12$, this equation has infinite real solutions.

$$
\begin{aligned}
2(x+12)-2 x & =3(x-2) \\
2 x+24-2 x & =3 x-6 \\
24 & =3 x-6 \\
-3 x & =-30 \\
x & =10
\end{aligned}
$$

Since $x=10$, the equation has one real solution.
14. Simplify the expression on the left side of the equation.

$$
\frac{4}{3}(-9 x+12)-x=-13 x+16
$$

An equation has exactly one solution when it has different coefficients on both sides of the equation. So, the coefficient of the $x$-term on the right side of the equation cannot be -13 . So, 13,11 , or -11 can be the coefficient of the $x$-term.

$$
\frac{4}{3}(-9 x+12)-x=13 x-9
$$

or

$$
\frac{4}{3}(-9 x+12)-x=-11 x-9
$$

or

$$
\frac{4}{3}(-9 x+12)-x=11 x-9
$$

An equation has no solutions when it has the same coefficient but different constants on both sides of the equation. So, the coefficient of the $x$-term on the right side of the equation must be -13 .

$$
\frac{4}{3}(-9 x+12)-x=-13 x+11
$$

An equation has infinitely many solutions when it has the same coefficient and constant on both sides of the equation. So, the coefficient of the $x$-term on the right side of the equation must be -13 and the constant must be 16 .

$$
\frac{4}{3}(-9 x+12)-x=-13 x+16
$$

15. Let $x$ be the number of guests.

Rick was charged a rental fee of $\$ 65$ and $\$ 25$ per guest. Write an expression to calculate the amount he spent for $x$ guests.

$$
25 x+65
$$

Tom was charged a rental fee of $\$ 40, \$ 25$ for music, and $\$ 25$ per guest. Write an expression to calculate the amount he spent for $x$ guests.

$$
25 x+40+25
$$

To find the number of guests that attended each party, create an equation using the two expressions, and solve for $x$.

$$
\begin{aligned}
25 x+65 & =25 x+40+25 \\
25 x+65-65 & =25 x+65-65 \\
25 x & =25 x \\
\frac{25 x}{25} & =\frac{25 x}{25} \\
x & =x
\end{aligned}
$$

As the solution to this equation is $x=x$, both sides of the equation will be true irrespective of the value substituted for $x$.

## Therefore, the same number of guests attended Rick and Tom's party.

